Effect of Isolated Tuned Dampers on Response of Multispan Structures

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An approximate analysis, utilizing the normal-mode method, is derived for the response of a multispan skin-stringer structure with isolated tuned viscoelastic dampers located at arbitrary points on the surface. The analysis is shown to lead to no coupling (or very little) between the modes of vibration of structures with identical panels when the dampers are placed at the center of each span and particularly simple solutions are derived in this case. To illustrate the technique, a solution is obtained for a specific idealized multispan structure.

Nomenclature

A_n = see Eq. (17)
D = $Eh^3/12(1 - \nu^2)$, flexural rigidity of skin
exp = exponential function
F_j = force transmitted back to structure by damper at point (x_j, y_j)
$F(x,y)$ = loading due to tuned dampers
i = $(-1)^{1/2}$
J = number of spans in structure, each span being bounded by stringers
j = span number
k = stiffness of tuned damper link
L = total length of panel array
l = breadth of panel between frames
m = mass of tuned damper (also dummy index)
N = number of modes in first frequency band
P_n = nth term in expansion of $P(x,y)$ as series of normal modes
$P(x,y)$ = amplitude of applied harmonic loading
S = see Eq. (8)
Sin = circular sine function
t = time
W = amplitude of transverse displacement of skin
W_n = nth term in expansion of W as series of normal modes
x = distance along structure parallel to frames
x_j = station of jth tuned damper along x axis
y = distance along structure normal to frames
y_j = station of jth tuned damper along y axis
Γ = kL^3/lD, stiffness parameter
Γ_e = effective stiffness parameter
δ = Dirac delta function; also y/l
Δ = x/L
η = loss factor of tuned damper link
μ = mass per unit area of panel
ξ = $(\mu\omega^2L^4/D)^{1/4}$, frequency parameter
ξ_D = $(\mu\omega_D{}^2L^4/D)^{1/4}$
ξ_n = $(\mu\omega_n{}^2L^4/D)^{1/4}$, nth eigenvalue of undamped structure
ϕ_n = nth normal mode of undamped structure
ψ = $Nm/\mu lL$, mass parameter
ψ_e = effective mass parameter
ω = frequency
ω_D = $(k/m)^{1/2}$, undamped natural frequency of tuned damper
ω_1 = lowest natural frequency of skin-stringer structure (stringer torsion mode)
ω_n = nth natural frequency of system
∇^4 = biharmonic operator

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I. Introduction

MANY complex structures exhibit multimodal response within definite frequency bands and excitation from jet engine, rocket engine, boundary layer or other sources often leads to failure or equipment malfunction. In such cases, surface strains in thin sheet metal structures are small, so that layered damping treatments are ineffective. On the other hand, vibrational amplitudes are often large and tuned dampers, with highly dissipative links, have been shown in the literature[1-3] to be effective over comparatively wide frequency bandwidths, leading to substantial reductions of response amplitudes over an entire frequency band. In this paper, approximate normal-mode analysis is developed for the response of a multispan skin-stringer-frame structure, under harmonic excitation, with tuned dampers attached.

II. Theory

A. Undamped Structure

The equation of free harmonic motion of a multispan structure, illustrated in Fig. 1, is the Euler-Bernoulli equation

$$D\nabla^4W - \mu\omega^2W = 0 \tag{1}$$

where W is the amplitude of the harmonic displacement at any point x,y. For free vibration of the undamped structure, it is possible to solve the homogeneous fourth-order differential equation for W in much the same way as for a simple beam. W will generally be zero except at discrete values ω_n of the natural frequency for which nontrivial solutions will exist. For a simply supported multispan beam, for example, a solution is given by Miles.[4] In general, for each of the ω_n, the solution may be expressed in the form $W = W_n\phi_n(x/L,y/l)$ where $\phi_n(x/L,y/l)$ is a function of x/L and

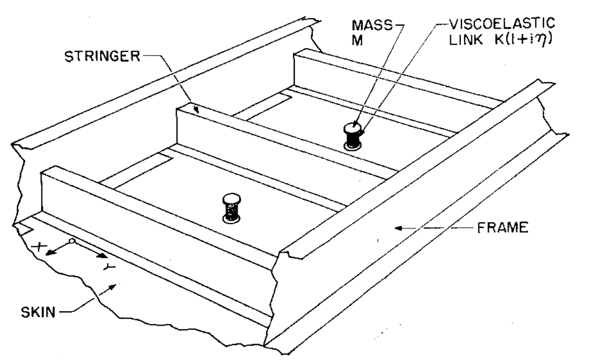

Fig. 1 Sketch of multispan skin-stringer structure with tuned dampers.

y/l which can be determined for given boundary and support conditions, and is normalized at some point, taken to be the point of maximum amplitude in this analysis. The function of $\phi_n(x/L,y/l)$ must satisfy the equation

$$\nabla^4\phi_n(x/L,y/l) = \xi_n{}^4\phi_n(x/L,y/l)/L^4 \tag{2}$$

where ξ_n in the nth eigenvalue of the system, also assumed to be known for each particular configuration.

For skin-stringer structures with uniform panel spacing and thickness and high stringer bending stiffness compared to the skin stiffness, Lin[5] has shown that the normal modes are grouped into well separated bands, the number of modes N in each band being equal to the number of spans J. This analysis is not restricted to uniform panel spacing or stringer stiffnesses provided that the normal modes and natural frequencies can be determined and the geometrical variations are not so great as to make the modal bands overlap. However, the skin thickness is assumed to be uniform.

B. Structure with Arbitrarily Located Tuned Dampers

Identical dampers are taken as located at points $x = x_j$, $y = y_j$ (j = 1 to J) along the structure. Obvious practical locations would be the center of each span but this is not essential to the analysis. Consider a tuned damper at the point $x = x_j$, $y = y_j$. The force F_j transmitted back to the structure at this point is readily shown[6] to be

$$F_j = \frac{m\omega^2 W(x_j,y_j)\exp(i\omega t)}{1 - m\omega^2/k(1 + i\eta)} \tag{3}$$

For all such dampers, the amplitude $F(x,y)$ of the load may be written

$$F(x,y) = -\frac{m\omega^2}{1 - m\omega^2/k(1 + i\eta)} \times \sum_{j=1}^{J} W(x_j,y_j)\delta(x - x_j)\delta(y - y_j) \tag{4}$$

In the analysis, it is now assumed that the normal modes $\phi_n(x/L,\ y/l)$ and the eigenvalues ξ_n are known for each particular structure in the absence of damping. We may then expand $F(x,y)$, $W(x,y)$, and $P(x,y)$ as series of these undamped normal modes. Since[5] the natural frequencies of multispan structures often fall into groups or bands, of which only the first is important, and since the number of modes in a band is equal to the number of spans N, we may write

$$W(x,y) = \sum_{n=1}^{N} W_n\phi_n\left(\frac{x}{L}\frac{y}{l}\right) \tag{5}$$

Table 1 Mode shape data $\phi_n(x/L)$ for five-span supported structure

	Symmetric mode no. n		
$5x/L$	1	3	5
0.0	0	0	0
0.1	+0.3044	−0.2883	+0.1610
0.3	+0.8090	−0.7226	+0.3645
0.5	+1.0000	−0.8090	+0.3090
0.7	+0.8090	−0.5414	+0.0745
0.9	+0.3044	−0.1384	−0.0679
1.0	0	0	0
1.1	−0.3044	+0.0398	+0.1926
1.3	−0.8090	−0.0948	+0.6642
1.5	−1.0000	−0.3090	+0.8090
1.7	−0.8090	−0.3880	+0.4850
1.9	−0.3044	−0.2028	+0.0511
2.0	0	0	0
2.1	+0.3044	+0.2637	+0.1506
2.3	+0.8090	+0.7812	+0.7103
2.5	+1.0000	+1.0000	+1.0000

The equation of motion for the structure with the dampers attached may be written in the form

$$D\nabla^4 W - \mu\omega^2 W + F(x,y) = P(x,y) \tag{6}$$

$$\nabla^4 W - \left(\frac{\xi^4}{L^4}\right)W - \left(\frac{S}{D}\right)\sum_{j=1}^{J}\delta(x - x_j)\delta(y - y_j) \times \sum_{m=1}^{N} W_m\phi_m\left(\frac{x_j}{L},\frac{y_j}{l}\right) = \frac{P(x,y)}{D} \tag{7}$$

where

$$S = \frac{m\omega^2}{1 - m\omega^2/k(1 + i\eta)} \tag{8}$$

Equation (7) may now be written

$$\sum_{n=1}^{N}(\xi_n{}^4 - \xi^4)W_n\phi_n\left(\frac{x}{L},\frac{y}{l}\right) - \left(S\frac{L^4}{D}\right) \times \sum_{=1}^{J}\delta(x - x_j)\delta(y - y_j)\sum_{m=1}^{N} W_m\phi_m\left(\frac{x_j}{L},\frac{y_j}{l}\right) = \sum_{n=1}^{N}\left(P_n\frac{L^4}{D}\right)\phi_n\left(\frac{x}{L},\frac{y}{l}\right) \tag{9}$$

where

$$P(x,y) = \sum_{n=1}^{N} P_n\phi_n\left(\frac{x}{L},\frac{y}{l}\right) \tag{10}$$

Using the orthogonal property of the normal modes, one may factor all terms of Eq. (9) by $\phi_n(x/L,y/l)$ and integrate with respect to x from 0 to L and with respect to y from 0 to l to give

$$(\xi_n{}^4 - \xi^4)W_n\int_0^l\int_0^L \phi_n{}^2\left(\frac{x}{L},\frac{y}{l}\right)dxdy - \left(P_n\frac{L^4}{D}\right)\int_0^L\int_0^l \phi_n{}^2\left(\frac{x}{L},\frac{y}{l}\right)dxdy - \left(S\frac{L^4}{D}\right)\sum_{j=1}^{J}\int_0^L\int_0^l \delta(x - x_j)\delta(y - y_j)\phi_n\left(\frac{x}{L},\frac{y}{l}\right)dxdy \cdot \sum_{m=1}^{N} W_m\phi_m\left(\frac{x_j}{L},\frac{y_j}{L}\right) = 0 \tag{11}$$

or

$$\left[(\xi_n{}^4 - \xi^4)W_n - P_n\frac{L^4}{D}\right]\int_0^l\int_0^L \phi_n{}^2\left(\frac{x}{L},\frac{y}{l}\right)dxdy = \left(S\frac{L^4}{D}\right)\sum_{j=1}^{J}\phi_n\left(\frac{x_j}{L},\frac{y_j}{l}\right)\sum_{m=1}^{N} W_m\phi_m\left(\frac{x_j}{L},\frac{y_j}{l}\right) \tag{12}$$

C. Structure with Dampers at Centers of Panels

The further evaluation of Eq. (12) demands a more detailed knowledge of the normal modes $\phi_n(x/L,\ y/l)$. For identical multispan supported beams, for example, the theory of Miles[4] has been used to determine the normal modes $\phi_n(x/L)$ and eigenvalues ξ_n. Here the variable y is not needed, and is dropped. The normal modes of a 9-span beam are given in Ref. 2 and those of a 5-span beam in Table 1. In these cases, it is readily shown that, for tuned dampers at the center of each span $J = N$, $x_j = (2j - 1)L/2N$ and

$$\sum_{j=1}^{N}\phi_n\left(\frac{x_j}{L}\right)\phi_m\left(\frac{x_j}{L}\right) = 0\ (m \neq n) \quad \doteq N/2\ (m = n) \tag{13}$$

For more complex structures with other support conditions,

$$\sum_{=1}^{N}\phi_n\left(\frac{x_j}{L},\frac{y_j}{l}\right)\phi_m\left(\frac{x_j}{L},\frac{y_j}{l}\right) \ll \sum_{j=1}^{N}\phi_n{}^2\left(\frac{x_j}{L},\frac{y_j}{l}\right) \tag{14}$$

Table 2 Values of A_n, ξ_n, and $\int_0^1 \phi_n(\Delta)d\Delta \Big/ \int_0^1 \phi_n^2(\Delta)d\Delta$ for five- and nine-span supported structures

No. of spans N	Mode no. n	$N\int_0^1 \phi_n(\Delta)d\Delta$	$N\int_0^1 \phi_n^2(\Delta)d\Delta$	A_n	ξ_n/N	ξ_n^4/N^4	$\int_0^1 \phi_n(\Delta)d\Delta \Big/ \int_0^1 \phi_n^2(\Delta)d\Delta$
5	1	+0.319	1.250	4.000	π	π^4	+0.255
	3	−0.754	1.210	4.132	3.700	187.4	−0.618
	5	+1.755	1.066	4.792	4.550	428.6	+1.647
9	1	+0.319	2.250	4.000	π	π^4	+0.141
	3	−0.668	2.236	4.020	3.345	125.2	−0.299
	5	+0.788	2.180	4.120	3.800	208.5	+0.362
	7	−1.133	2.046	4.400	4.298	341.1	−0.554
	9	+3.050	1.833	4.900	4.670	475.8	+1.664

and this criterion is necessary for the valid neglect of all terms of the summation with respect to j in Eq. (12) apart from the term $m = n$. Therefore Eq. (12) becomes

$$\left[\xi_n^4 - \xi^4 - \left(S\frac{L^3}{D}\right)\sum_{j=1}^{N}\phi_n^2(\Delta_j,\delta_j)\Big/\int_0^1\int_0^1 \phi_n^2(\Delta,\delta)d\Delta d\delta\right] W_n = P_n\frac{L^4}{D} \quad (15)$$

$$\therefore W_n = \frac{P_nL^4/D}{\xi_n^4 - \xi^4 - (SL^3A_n/lD)} \quad (16)$$

where

$$A_n = \sum_{j=1}^{N}\phi_n^2(\Delta_j,\delta_j)\Big/\int_0^1\int_0^1 \phi_n^2(\Delta,\delta)d\Delta d\delta \quad (17)$$

But

$$SL^3\frac{A_n}{lD} = \frac{m\omega^2L^4A_n/lD}{1 - m\omega^2/k(1+i\eta)} = \frac{\psi A_n\xi^4}{1 - \psi\xi^4/\Gamma(1+i\eta)} \quad (18)$$

so that, finally,

$$W = \left(\frac{L^4}{D}\right)\sum_{n=1}^{N}\frac{P_n\phi_n(x/L,y/l)}{\xi_n^4 - \xi^4 - \dfrac{\psi A_n\xi^4}{1 - \psi\xi^4/\Gamma(1+i\eta)}} \quad (19)$$

Equation (19) may be written in the alternative form

$$\frac{DW}{L^4} = \sum_{n=1}^{N}\frac{P_n\phi_n(x/L,y/l)Z_{rn}}{Z_{rn}^2 + Z_{in}^2} + i\sum_{n=1}^{N}\frac{P_n\phi_n(x/L,y/l)Z_{in}}{Z_{rn}^2 + Z_{in}^2} \quad (20)$$

where

$$Z_{rn} = \xi_n^4 - \xi^4 - \frac{\psi\Gamma A_n\xi^4[\Gamma - \psi\xi^4 + \eta^2\Gamma]}{(\Gamma - \psi\xi^4)^2 + \Gamma^2\eta^2}$$

$$Z_{in} = \frac{\eta\psi^2A_n\Gamma\xi^8}{(\Gamma - \psi\xi^4)^2 + \Gamma^2\eta^2}$$

As a further approximation, it will be noted that for multispan supported structures (Table 2) A_n is nearly 4.0 for all n and one may define an effective mass ratio ψ_e and effective stiffness ratio Γ_e such that $\psi_e = 4\psi$ and $\Gamma_e = 4\Gamma$. In this case, approximately

$$\left(\frac{DW}{L^4}\right) = \sum_{n=1}^{N}\frac{P_n\phi_n(x/L,y/l)}{\xi_n^4 - \xi^4 - \dfrac{\psi_e\xi^4}{1 - \psi_e\xi^4/\Gamma_e(1+i\eta)}} \quad (21)$$

This means that, as an approximation, the theory of distributed tuned dampers on the multispan structure may be used without change, beyond replacing ψ by ψ_e and Γ by Γ_e. The method of analysis is unchanged and response spectra may be obtained numerically for given loading $P(x,y)$.

D. Example

Since data concerning mode shapes and natural frequencies of multispan skin-stringer structures are not generally available in the literature, an idealized situation will be considered. Consider a multispan supported structure with uniform stringer spacing, infinite stringer bending stiffness, and pinned along the frames, so that the mode shape in the y direction will be $\text{Sin}(\pi y/l)$. It is shown by Mercer[7] that, if the panel aspect ratio Nl/L is greater than about 2, the equation of motion in the x direction, parallel to the frames, reduces to the beam equation. Miles' analysis[4] therefore may be applied to the determination of the mode shapes in the x direction. Mode shapes for 5- and 9-span beams are given in Table 1 and Fig. 2.

On the basis of these modes, the values of A_n can be calculated. These are given in Table 2 along with those for a 9-span structure of this type, based on data from Ref. 2. The values of P_n/P are also readily obtained from Eq. (10). For uniform loading P

$$\frac{P_n}{P} = \int_0^1 \phi_n(\Delta)d\Delta\int_0^1 \text{Sin}(\pi\delta)d\delta\Big/\int_0^1 \phi_n^2(\Delta)d\Delta\int_0^1 \text{Sin}^2(\gamma\delta)d\delta \quad (22)$$

$$= \left(\frac{4}{\pi}\right)\int_0^1 \phi_n(\Delta)d\Delta\Big/\int_0^1 \phi_n^2(\Delta)d\Delta$$

and is $4/\pi$ times the value of P_n/P for a multisupported beam. Calculated values of the integrals involved in Eq. (22) are given in Table 2 for odd values of n. P_2/P and P_4/P are zero by symmetry. The expression for DW/PL^4 for this panel system with tuned dampers at the centers of

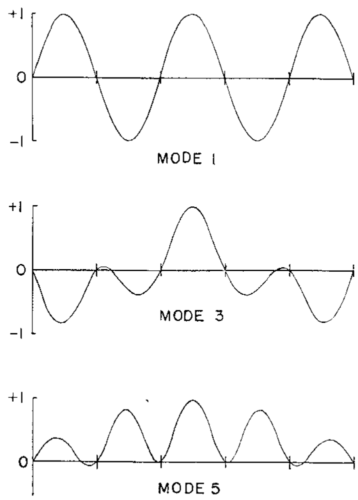

Fig. 2 Symmetric mode shapes for 5-span supported structure.

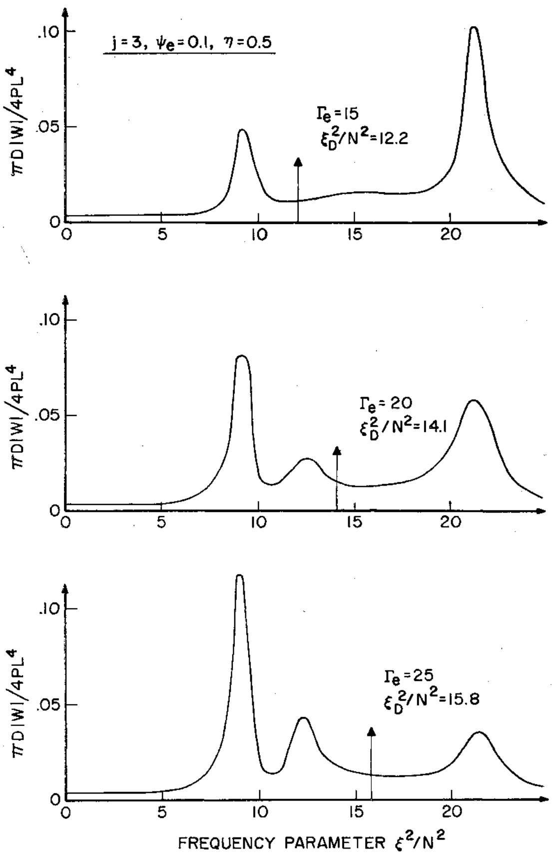

Fig. 3 Typical response spectra for 5-span supported structure with tuned dampers.

each panel, given by Eq. (21), is readily evaluated using a digital computer for given ψ_e, η, Γ_e, and j. Typical graphs of $(\pi D/4PL^4)|W|$ at the center of the center panel ($j = 3$) against ξ^2/N^2 are shown in Fig. 3. The value of ξ^2/N^2 to which the dampers are tuned is given[2] by $\omega_D = \omega_1(\Gamma_e/\psi_e\xi_1^4N^4)^{1/2}$ so that $\xi_D^2/N^2 = (\Gamma_e/\psi_e)^{1/2}$. The points to which the dampers are tuned are shown in Fig. 3. The effect of changing the damper tuning frequency on the response can be clearly seen.

Conclusions

A theory has been developed for the determination of the effect of isolated tuned dampers on the response of a multi-span skin-stringer structure to a harmonic loading with arbitrary spatial dependence. It is shown that, for panels with identical or nearly identical stringer separations, the normal modes are almost decoupled when identical isolated dampers are placed at the center of each span. An example is given.

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